The paradox of adversarial liquidation in decentralized lending

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Decentralized lending

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Lending is the basis of financial systems

- A key part of the DeFi ecosystem
- Large values currently lent – more than 30% TVL in Ethereum's protocols
- Different from classical finance, as no recourse (e.g. to the courts, credit ratings, ...)
- Risk management is critical

Collateral rules

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How do we protect against bad debt?

- As there is no recourse, positions need to be overcollateralized.
- We wish to borrow b units of asset X, using c units of Y as collateral, with P as price of Y in numeraire X.
- Our collateral value is cP, and we assign a haircut factor θ to give a maximum borrowed amount $\theta c P_t$
- The *health factor* of our position is then $\mathbb{HF}_t = \frac{b}{\theta c P_t}$. Our position is in margin-default if $\mathbb{HF}_t \leq 1$.
- A higher value of θ is often chosen to establish a position (cf. initial vs maintenance margin)
- We choose θ based on a variety of principles, as we shall see.

(Partial) Liquidation

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- When a position is in margin-default, any participant is able to close-out the position.
- This differs from classical clearing, where this is a role of the clearing house, who do not typically trade speculatively on their own account.
- The liquidator repays the loan value (b units of X), and receives the collateral value of the position, plus a proportional bonus, i.e. $(1+\ell)b/P_t$ units of Y
- The reward ℓ is to encourage liquidators to act, and is paid from the overcollateralization of the position.
- Some protocols also limit the fraction of a position which can be liqudiated in a single transaction.

(Partial) Liquidation

- In order to ensure a position can be fully closed out, we require $(1+\ell)b/P_t \leq c$, which simplifies (as $c = \theta b P_t$) to $\theta(1+\ell) \leq 1$.
- This gives us a bound between the rewards and the collateralization level, in particular an upper bound on ℓ.
- In general (with partial liquidation), the health factor of a position will improve after liquidation iff $\mathbb{HF}_t > \theta(1+\ell)$
- We can then choose θ to ensure an expected-shortfall type condition is preserved, to avoid the risk to liquidity providers if liquidators do not act quickly.

Enabling Liquidation

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- As liquidation is done by general agents (not the protocol), they will only act if it is profitable to do so.
- As they have to expend X and receive Y, we need to account for the cost of reversing this transaction.
- This gives us a basic guide to a lower bound on ℓ.
- We assume that the liquidator will immediately reverse their trade, trading an amount y for x. We suppose they face a price $P_t - \Delta(P_t, x)$ for this trade, and move the price to $P_t - H(P_t, x)$.
- If the trading is in an AMM, these quantities are known and computable.

Enabling Liquidation

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If the liquidator liquidates a fraction κ of the loan, and minimally trades of offset their position, we have the sequence of cashflows

1. Liquidate: $-\kappa b$ units of X and $+(1+\ell)\kappa b/P_t$ units of Y
2. Trade: $+\kappa b$ units of X and $-\kappa b/(P_t - \Delta(P_t, \kappa b))$ units of Y

Net position in Y:

$$\kappa b\left(\frac{(1+\ell)}{P_t} - \frac{1}{P_t - \Delta(P_t, \kappa b)}\right)$$

This is a profit iff $1+\ell > \frac{P_t}{P_t - \Delta(P_t,\kappa b)} = \left(1 - \frac{\Delta(P_t,\kappa b)}{P_t}\right)^{-1}$.

Enabling Liquidation

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The net result of this is that, in order to have the risk-management system operating properly, without assuming liquidators will bear market risk, we need

$$\frac{1}{1+\ell} \in \left[\theta, \quad \left(1 - \frac{\Delta(P_t, \kappa b)}{P_t}\right)\right]$$

- This ties the functioning of the liquidation system to the liquidity of the reference market.
- Low liquidity in the market makes risk management more difficult.
- Usually θ, ℓ will be fixed for longer periods, leading to a potential market failure.

Protocol risks

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A lending protocol faces a variety of practical risks

- Bank runs – particularly if collateral is rehypothecated for lending (which is needed if interest is to be paid on collateral)
- Wrong way risk – failures occur when one asset collapses
- Adverse selection and arbitrage – if θ is low, bad debt may be cheaper than purchasing assets directly
- Liqudiation spirals
- Adversarial liquidation and short squeezes

We will focus on the final two of these.

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- Our earlier liquidation model assumes liquidators are largely passive, as in traditional clearing.
- However, here they have the ability to front-run the liquidation process.
- As price impact is known (when the reference/oracle market is an AMM), this causes problems...

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An adversarial liquidator can act as follows:

- Identify a loan with health factor $\mathbb{HF}_t \leq (1 - \frac{H(P_t,\kappa b)}{P_t})$

1. Trade: $+\kappa b$ units of X and $-\kappa b/(P_t - \Delta(P_t, \kappa b))$ units of Y, moving the price to $P_t - H(P_t, \kappa b)$.

- Notice that this moves $\mathbb{HF}$ below 1, and hence the position can be liquidated

2. Liquidate: $-\kappa b$ units of X for $+\frac{(1+\ell)\kappa b}{P_t - H(P_t,\kappa b)}$ units of Y.

Net position:

$$\kappa b\left(\frac{1+\ell}{P_t - H(P_t, \kappa b)} - \frac{1}{P_t - \Delta(P_t, \kappa b)}\right)$$

Adversarial Liquidation

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This leads to the paradox of adversarial liquidation:

- In order for passive liquidation (without price manipulation) to be profitable, we require

$$1 + \ell > \frac{P_t}{P_t - \Delta(P_t, \kappa b)}$$

- But this implies, under reasonable market assumptions,

$$1 + \ell > \frac{P_t - H(P_t, \kappa b)}{P_t - \Delta(P_t, \kappa b)}$$

  and we see that frontrunning the trade is more profitable.

Practically, this implies the critical health factor is above 1, but the reward to liquidators ℓ could be lowered.

Liquidity at risk

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- For a protocol with rehypothecation of collateral, a key concern is liquidity at risk – how much collateral will be demanded by liquidators in the short run?
- This depends on whether liquidators front-run trades or not.
- We define the function $\mathcal{L}(p) = \frac{\kappa(1+\ell)}{p} \sum_j b^j 1_{\{\theta c^j p \leq b^j\}}$, which describes the quantity of Y demanded if the price moves to p.
- Without front running, one simply computes the expected shortfall of $\mathcal{L}(P_{t+h})$ over the desired horizon

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- With front running, we assume that liquidators will manipulate the market as much as it is profitable to do so.
- We then compute the maximum amount in x which liquidators will want to trade, given their price impact:

$$\mathcal{X}(p) = \arg\max_x \left\{ p\mathcal{L}(p - H(p, x)) - x \right\}$$

and hence the liquidation at risk accounting for front running

$$\mathcal{L}(p - H(p, \mathcal{X}(p)))$$

- The expected shortfall can then be computed via simulation, as usual.